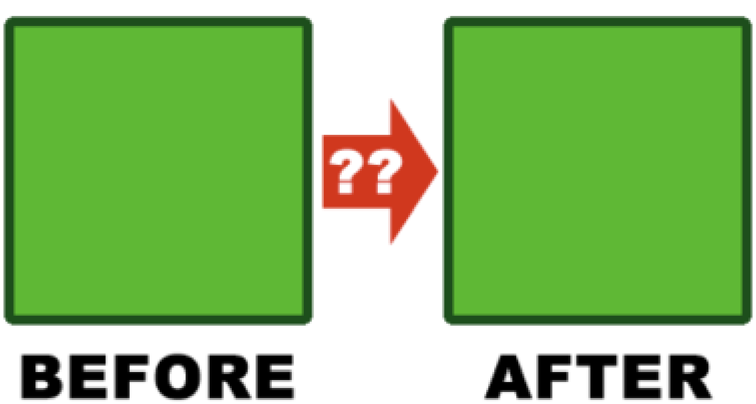
SYMMETRY v.1 (square)

What is "symmetry"? I can't remember. Help!

SQUARE PUZZLE

My teacher loves puzzles—I do too!
Here is the latest one: "How did the square change?"



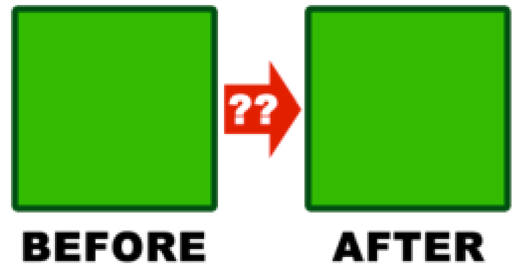
I don't see a change. Do you have any ideas?

SYMMETRY DEFINITION

Did you know mathematicians see symmetry as a transformation!

> A symmetry is a transformation that leaves an object looking unchanged.

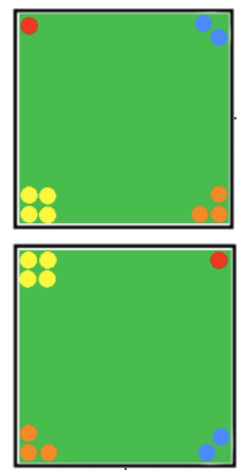
Now I get the puzzle!



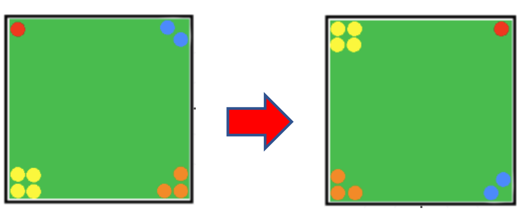
ROTATION SYMMETRIES OF THE SQUARE

I have an idea! Let’s label the vertices of the square with 1, 2 ,3 and 4 dots, to see the transformations.

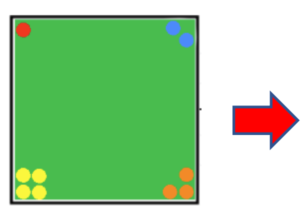
Rotate the square ¼ turn (or 90 degrees) clockwise. The order of the dots will change: from **1234** to **4123**.



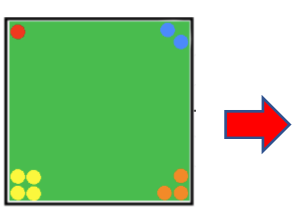
I wonder how many **rotation symmetries** a square has. My guess is 4, since it has 4 sides. Here’s one of them. Can you find the rest?



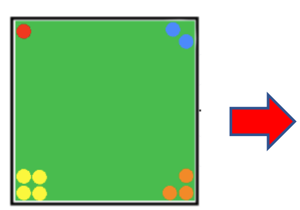
Rotate ¼ or **90** degrees



Rotate _____ or _____ degrees

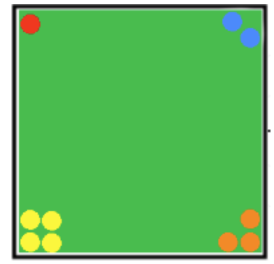


Rotate _____ or _____ degrees



Rotate _____ or _____ degrees

PERMUTATIONS



I learned a new math word: **permutation**!
A permutation is an arrangement.

We found 4 permutations of the numbers **1**, **2**, **3** & **4** by rotating the square: **1234**, **4123**, **3412** & **2341**.

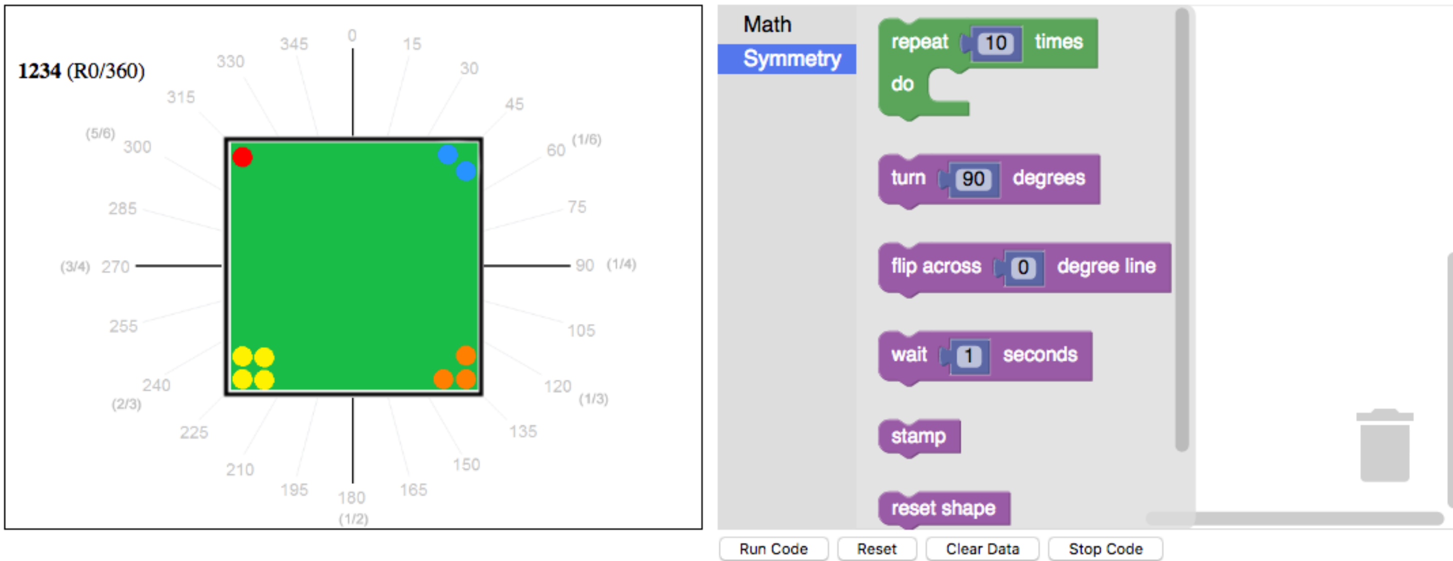
But there are more ways to arrange these numbers—right?
Help me find them all—it will be fun!

Let's use an organized list, not to miss any—do you agree?

And, let's circle the 4 rotation permutations.

ROTATE THE SQUARE WITH CODE

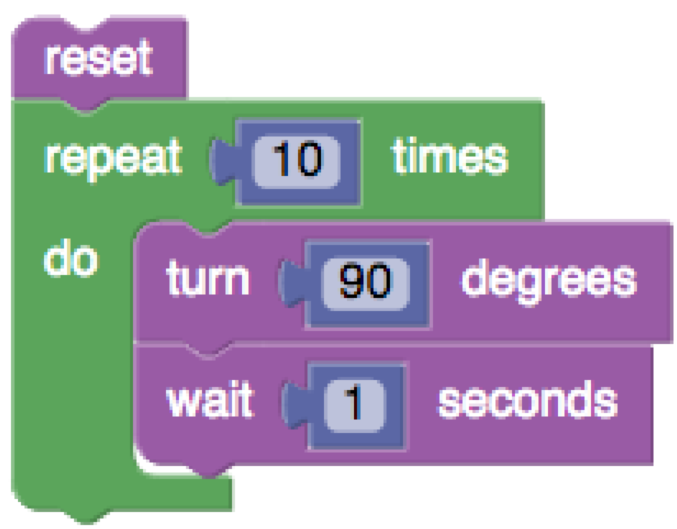
Wow! We can use coding for transformations! Want to try it?
Go to researchideas.ca/sym/shapesTurnFlip.html



TIP: *If this view does not fit on your screen, press Control (or Command) and minus (-) to reduce screen resolution.*

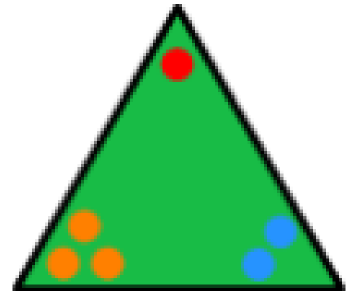
Select the square.
Enter this code.
Then click Run Code.

Notice the rotation permutations listed on the left of the screen.



Is it possible to get permutations other than **1234**, **4123**, **3412** & **2341** by rotating the square? What do you think?

ROTATE THE TRIANGLE WITH CODE

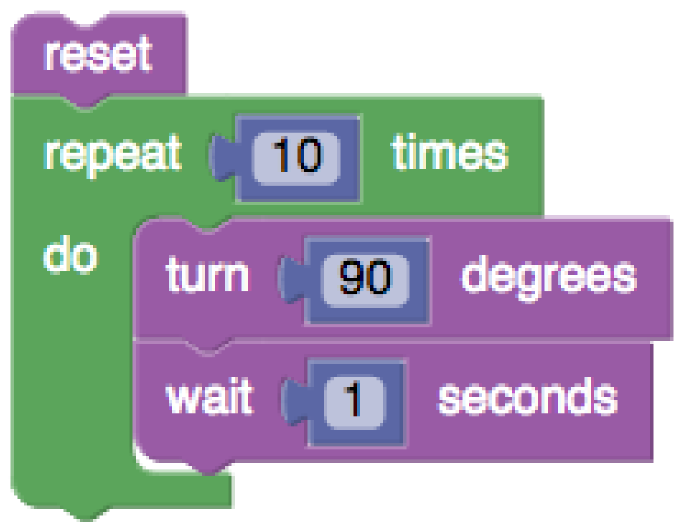


Hey! I want to rotate the triangle too! Don't you?

The square has 4 rotation symmetries, with permutations 1234, 4123, 3412 & 2341.

What about the triangle? Help me predict:

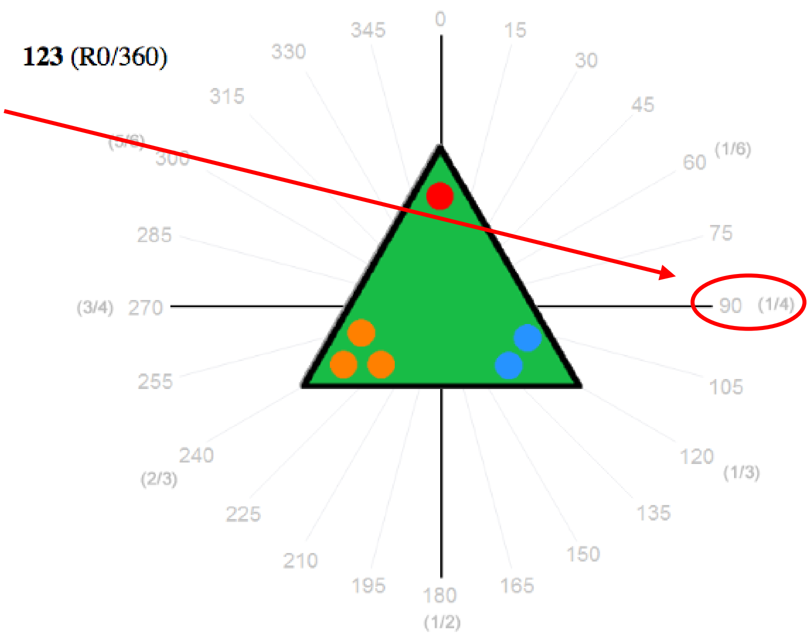
- How many rotation symmetries does the triangle have? _____
- What are the permutations? 123, _______, _______



O.K. Let's code. Select the triangle in the coding environment.

Here is the code we used to rotate the square. Hmmm. How many degrees should we rotate the triangle, so that it looks unchanged?

Stuck? Here's a hint: look at the fraction turns.



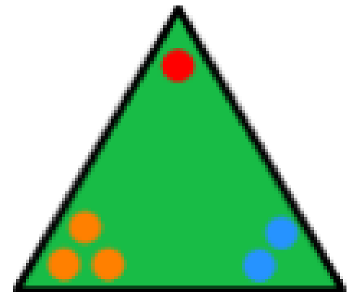
For the square, it was 1/4 turn, or 90 degrees.

Should the triangle rotate 1/6, 1/4 or 1/3 of a turn?

What's the matching degree measurement?

Got it?

PERMUTATIONS



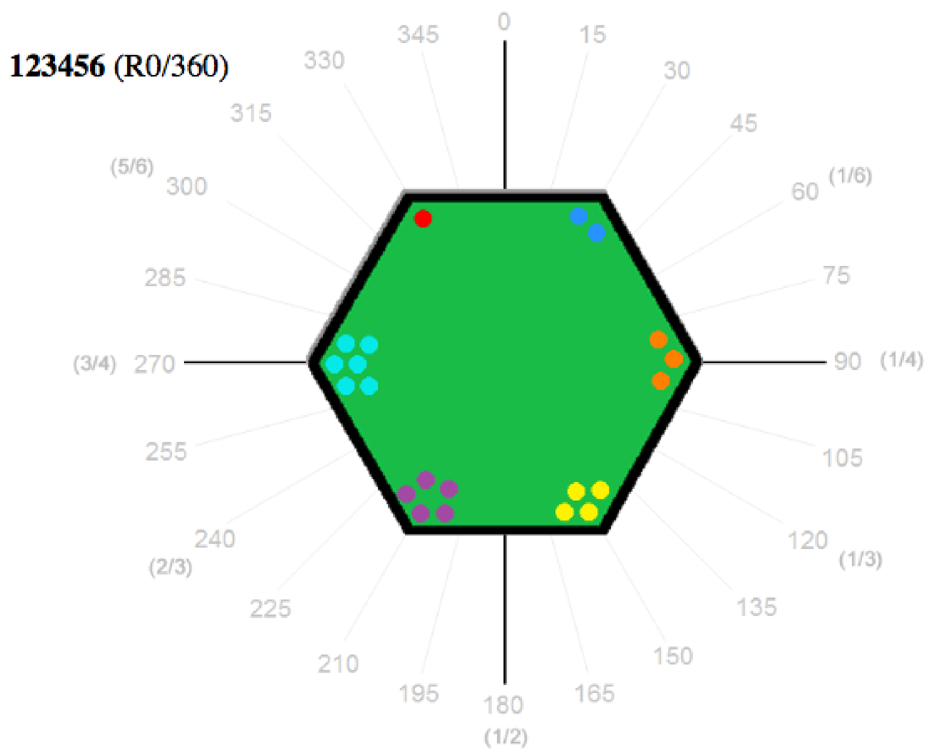
I wonder. There were 24 permutations for the numbers 1, 2 3, & 4. How many different permutations are there with the numbers 1, 2 & 3?

Can you help me find them? Of course you can!

ROTATE THE HEXAGON WITH CODE

Bees love hexagons—and so do I!
How many rotation symmetries does the hexagon have? _____

What will the rotation permutations be?
Help me list them.



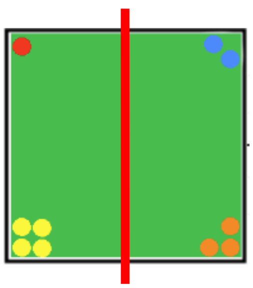
How many degrees do we rotate the hexagon, so it looks unchanged?

Give it a try with the coding environment.

REFLECTION SYMMETRIES OF THE SQUARE

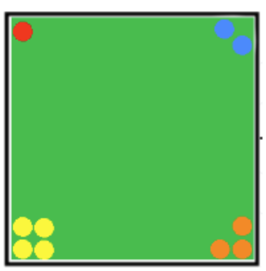
Did you know that "flip" and "reflect" is the same transformation?

For example, we can flip, or reflect, the square across its vertical line of symmetry. Do you see how the permutation changes?

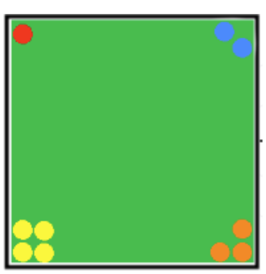


1234 ⇨ ______

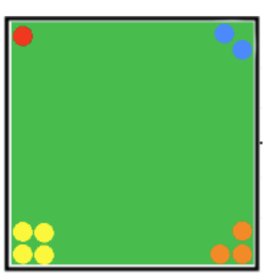
I think there are 3 more ways. Help me draw the remaining lines of symmetry and record the permutations.



1234 ⇨ ______



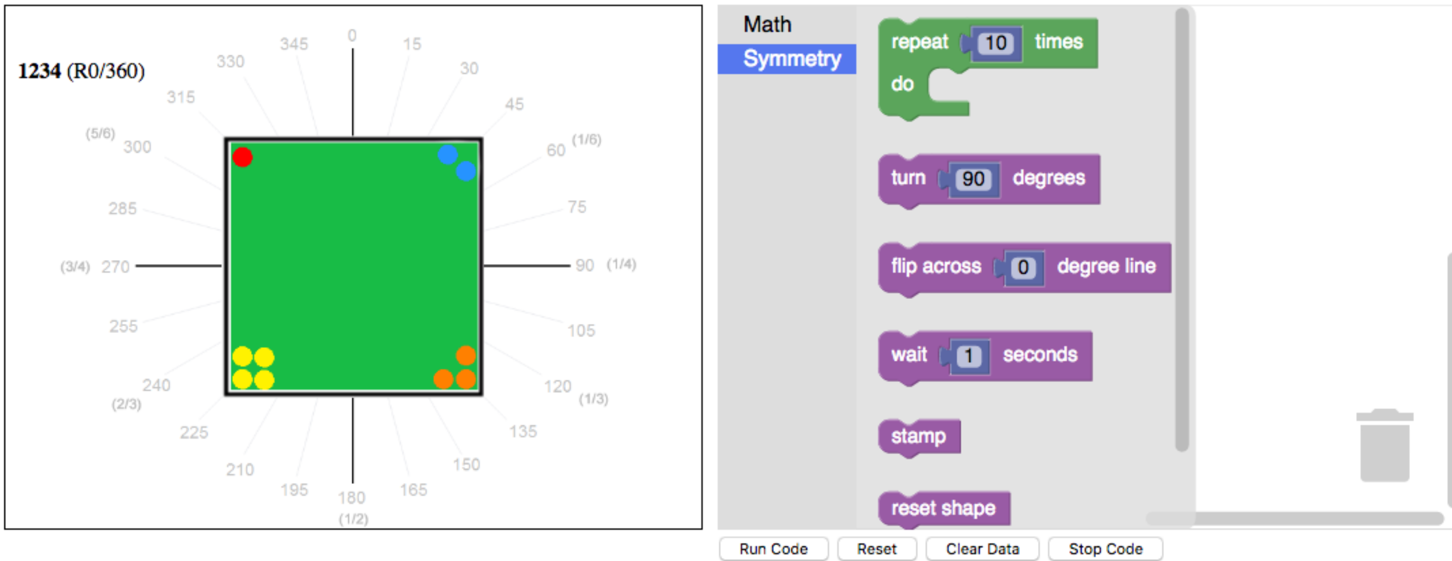
1234 ⇨ ______

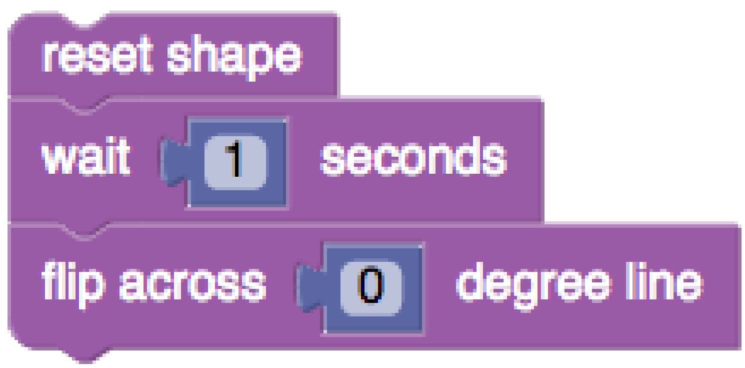


1234 ⇨ ______

REFLECT THE SQUARE WITH CODE

Hey! We can use code for reflections too! Want to try it?
Go to researchideas.ca/sym/shapesTurnFlip.html





Select the square. Enter this code.
Then click Run Code.

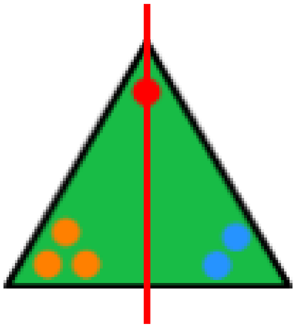
The **0 degree line** is the vertical line of symmetry. What are the degrees for the other lines of symmetry?

For each line of symmetry, set the degrees in the code, and then Run Code. Record the permutation for each reflection.
Do they match the permutations you found on the previous page?

On page 3, underline permutations that match the reflection symmetries of the square.

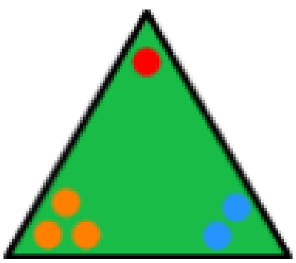
REFLECTION SYMMETRIES OF THE TRIANGLE

What are the reflection symmetries of the triangle?
Here's one. Do you see how the permutation changes?

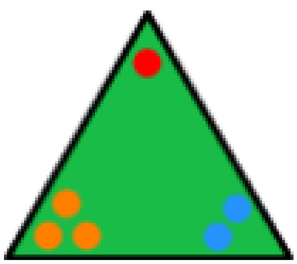


123 ⇨ ______

Two more, I think. Draw lines of symmetry & record permutations.



123 ⇨ ______



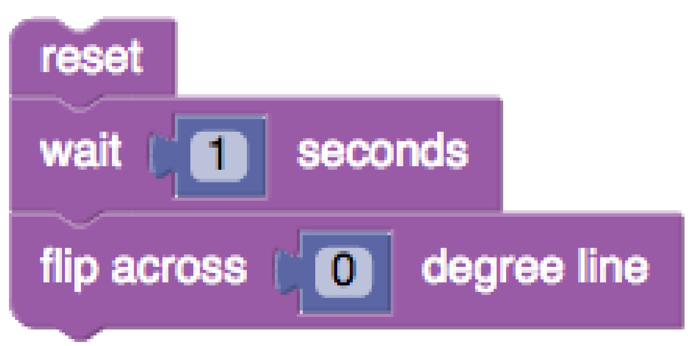
123 ⇨ ______

REFLECT THE TRIANGLE WITH CODE

Let's try this with code!
Go to researchideas.ca/sym/shapesTurnFlip.html
Select the triangle.

The **0 degree line** is the vertical line of symmetry.

What are the degrees for the other lines of symmetry?



COMBINE SQUARE REFLECTIONS WITH CODE

I made a great discovery!
You'll be amazed!

Go to researchideas.ca/sym/shapesTurnFlip.html

Enter this code.
Then click on Run Code.

```
reset
wait 1 seconds
flip across 0 degree line
wait 1 seconds
flip across 45 degree line
```

Do you see what I mean?
We flip the square across the 0 degree line.
Then we flip it across the 45 degree line.
And the permutation is **4123**!

Do you see why this is amazing?
Need a tip?
Go to page 3 and see if **4123** is circled or underlined.

Let's try more combinations.
Change **0** and **45** degrees to any pair of the following:

0, 45, 90, 135, 180
225, 270, 315, 360
-45, -90, -135, -180
-225, -270, -315, -360

```
reset
wait 1 seconds
flip across 0 degree line
wait 1 seconds
flip across 45 degree line
wait 1 seconds
flip across 90 degree line
```

Repeat with different pairs.

Also, try 3 flips in a row!

What do you notice?
Any more surprises?

MATH DISCOVERY

We made an important math discovery! Don't you agree?

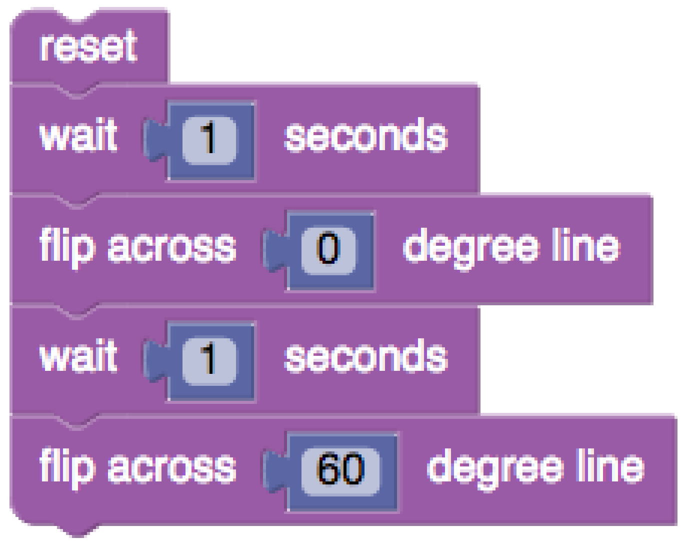
For squares, rotations are a different *species* than the reflections:

- When we combine rotations, we always get another rotation.
- But, when we combine reflections, sometimes we get a reflection and sometimes a rotation.

COMBINE TRIANGLE REFLECTIONS WITH CODE

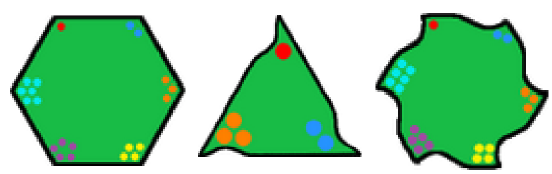
I bet you're wondering, "What about the triangle?"
Me too!

Let's combine triangle reflections.



MORE SHAPES

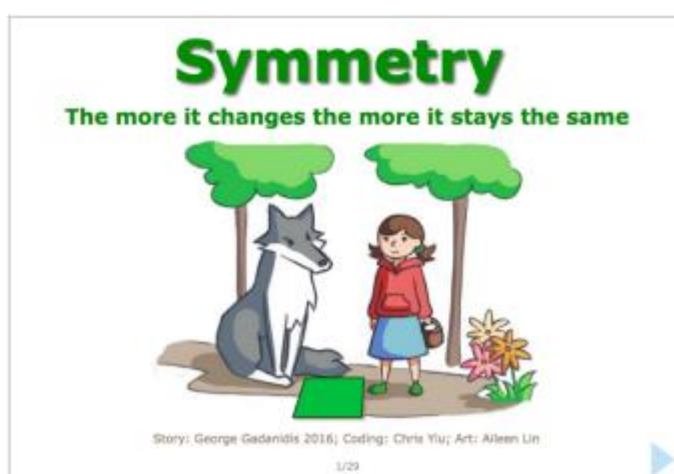
Let's not forget the hexagon!
And the other shapes, too!
What are their symmetries?
What happens when we combine them?



MATH STORY

Read a symmetry story at:
researchideas.ca/sym/story.html

Symmetry: the more it changes the more it stays the same



REFLECT

I learned so much! How about you?

What did you learn?

What else do you want to know or explore?