WHY CAN'T I BE A MATHEMATICIAN?

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Looking in the mirror of society to explain why children cannot be (or would not want to be) mathematicians, we might see two unflattering images: "math sucks" and "mathematicians are geeks". Of course, mathematics is beautiful and mathematicians are cool, but these appear to be secrets that we, the mathematics community, somehow keep to ourselves. How might we share the beauty of mathematics and the coolness of being a mathematician with a wider audience?

At a symposium in 2004, Apostolos Doxiadis, a mathematics prodigy who eventually became a filmmaker and author, said that mathematics education will not change unless what counts as mathematics first changes—what counts as mathematics needs to include the *stories* of doing mathematics. [1]

Story and audience form the cornerstones of the mathematics classroom work discussed in this article. Put in the simplest terms, our starting point for collaborative lesson planning is to imagine a good mathematics story to be shared by our students with family and friends and with the wider community: a story that will offer a new and wonderful perspective on a mathematical concept, that will create an opportunity for mathematical surprise, that will engage emotionally, and will offer a sense of mathematical insight and beauty. These stories exist and can be shared by and for mathematicians of all ages, if we are willing to (a) cross arbitrary grade boundaries with low floor, high ceiling mathematics so that young students may explore such topics as infinity and limit, and (b) engage ourselves and our students in arts-based mathematics communication and the pleasure of solving artistic puzzles in mathematical storytelling.

Defining the problem: mathematics

Papert (1980) states, "Children begin their lives as eager and competent learners. They have to *learn* to have trouble with learning in general and mathematics in particular" (p. 40). What is it about the school mathematics experience that turns eager mathematics learners into mathematics avoiders? Higginson (2008) calls it "*smath*, it's school math, it's a very strange variant on mathematics, and I don't like it very much." Higginson elaborates that school mathematics is not about "the power of mathematical ideas" or "the beauty of mathematical concepts."

There are powerful and beautiful mathematical ideas that have intrigued mathematicians across history which have the potential to capture young students' intellectual interest and imagination, such as, for example, Zeno's paradoxes concerning infinity and limit, Euclid's Parallel Postulate and non-Euclidean geometries, and Descartes' bridging of algebra and geometry. However, in mathematics curricula, it seems that such ideas are somehow not appropriate for young children. But why do we think that young children might not be ready for concepts such as these? Or, putting it another way, why do we have such faith in the curriculum sequence that we have created? I think part of the answer lies in the legacy left to us by the work (or the interpretation of the work) of Jean Piaget, and our resulting emphasis on young children being concrete thinkers and not ready for abstract concepts.

Piaget's stages of cognitive development, which were developed in an era when there was a paramount focus on the classification of individuals, dominate in primary education (Egan, 2002; Walkerdine, 1984). Although Piaget's constructivism has made a significant contribution to mathematics reform by showing that "learning originates from *inside* the child" (Kamii & Ewing, 1996, p. 260), his work on developmental stages misrepresents children's potential. Piaget (2008/1972) himself expresses some caution about how generally his stages of development might apply, noting that:

> we used rather specific types of experimental situations [...] it is possible to question whether these situations are, fundamentally, very general and therefore applicable to any school or professional environment. (p. 46)

Papert (1980) challenges the linear progression of Piaget's developmental stages and suggests the sequence identified by Piaget is not in children's minds but in the "poverty of the culture" of schooling (p. 7). Piaget (2008/1972) agrees, to the extent that "the rate at which a child progresses through the developmental succession may vary, especially from one culture to another" (p. 40). Egan (2002) suggests that children "are not designed [...] to move in the direction of 'formal operations' or abstract thinking or whatever. These forms of intellectual life are products of our learning" (p. 114). Moreover, Fernández-Armesto (1997) argues that "generations of school children, deprived of challenging tasks because Piaget said they were incapable of them, bear the evidence of his impact" (p. 18). In this context, I seek to disrupt the common pedagogical assumption that students learn best when *learning is made easy* by designing activities where they have opportunities to *think hard.*

Defining the problem: mathematicians

Mathematicians are, according to Picker and Berry (2000) "essentially invisible, with the result that pupils appear to rely on stereotypical images from the media to provide images of mathematicians" (p. 88). These images are quite consistent: "popular discourses overwhelmingly construct mathematicians as white, heterosexual, middle-class men, yet also construct them as 'other' through systems of binary oppositions between those doing and those not doing mathematics" (Moreau, Mendick & Epstein, 2010, p. 10). This

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portrayal raises important issues: about how popular culture may be deterring many people from enjoying and continuing their study of mathematics; and about social justice, since the images of mathematicians as mainly white, middle-class men may discourage other groups disproportionately (Moreau, Mendick & Epstein, 2010). Piatek-Jimenez (2008) found that stereotypical beliefs about mathematicians are also held by undergraduate women mathematics students. Although women earn nearly half the mathematics baccalaureate degrees in the United States, many fewer carry on with further study or mathematics oriented careers: "their beliefs about mathematicians may be preventing them from identifying as one and choosing to pursue mathematical careers" (p. 633). Narrow, negative images of mathematicians appear to overlap and likely have a bi-directional relationship with images of mathematics, which is often viewed as "difficult, cold, abstract, and in many cultures, largely masculine" (Ernest, 1996, p. 802).

There is also an *adult* bias in students' depictions of mathematicians. Although the labeling of students as "mathematicians" is not uncommon in the mathematics literature, there is no evidence to suggest that students see themselves or their peers as mathematicians, or "young" mathematicians. In the same way that we might not want students to simply associate "athlete" with professional sport and "artist" with people who produce museum art, we do not want students to associate "mathematician" simply with professional mathematicians, or for "a talented few" (Henderson, 1981). To paraphrase Dissanayake's (1992, p. 33) view of art, we need students to see mathematics as a "normal, natural and necessary" part of human experience, for both young mathematicians and professional mathematicians.

A wider audience for mathematics

What might change in classrooms to help students, teachers and the public in general, experience more robust images of mathematics and mathematicians? The approach of my collaborative work [2] with teachers in elementary school classrooms is four-fold. First, we engage students with activities that have a low mathematical floor, allowing engagement with minimal prerequisite mathematical knowledge; and a high mathematical ceiling, so that concepts and relationships may be extended to more complex connections and more varied representations (Gadanidis & Hughes, 2011). Typically, we start with a concept from the secondary school mathematics curriculum, and we design low floor, high ceiling activities that would engage Grades 2-4 students (aged 7-10 years).

Second, we use children's literature to introduce, reinforce or extend these mathematics ideas. When we cannot find an existing story to match the concepts we plan to teach, we write our own. The vicarious connections that students make with the mathematical predicaments of the characters in a story create a much more emotionally intense and, hence, mathematically more engaging and memorable learning experience (Whittin & Gary, 1994; Whittin & Wilde, 1995). Children's literature also models mathematical storytelling for students and creates opportunities for them to retell and extend stories.

Third, we help students to develop arts-based communication skills for relating stories of their learning to family, friends and the wider community. Working in small groups, students script dramatic dialogues to share their learning. This sharing may be seen as a form of community service (Hughes & Gadanidis, 2010), which enhances the sense of audience, motivating and giving purpose to students' learning; creates school-community links by opening public windows into school learning; and provides an opportunity for students to give voice to their mathematical identity. When students are given opportunities to share their "identity texts" with peers, family, teachers and the general public, they are likely to gain in self-confidence, self-esteem and a sense of community belonging through positive feedback (Cummins, Brown & Sayers, 2007, p. 219). Furthermore, Hull & Katz (2006) note "the power of public performance in generating especially intense moments of self-enactment" (p. 47). There is also a dialogical relationship between narrative/identity and community: narratives are social artefacts and "the narrated self is constructed with and responsive to other people" (Miller, 1994, p. 172). Burton (1999) notes that "learning mathematics as a narrative process where the learners have agentic control over authorship makes a substantial difference" in terms of achievement and attitude towards mathematics (p. 31).

Fourth, we use student writing and shared thinking to write songs that capture and celebrate the collective knowledge of the class. In some cases, songs are written by the students. Student performances of these songs are typically performed in other classrooms and shared publicly through the Math Performance Festival. [3] They are also performed in school concerts across Ontario. [4] In some cases we also create artistic renderings of the learning experience as posters (see Figure 1) to be shared with other schools, with the original works of art donated to project schools.

Documentaries of this approach can be found at www.researchideas.ca, on such topics as limit and infinity, linear functions, optimization, non-Euclidean geometry, sequences and series, and trigonometry. Below I elaborate on our Grade 3 approach to the concepts of infinity and limit which, although historically difficult for mathematicians to conceptualize (Kline, 1980; Kleiner, 2001), can be explored by "mathematicians" of a wide range of ages and mathematical sophistication (*e.g.,* Boero, Douek & Garuti 2003; Tall & Tirosh, 2001).

Infinity and limit in Grade 3

We engage Grade 3 students with infinity and limit by posing the question, "Can we walk out the classroom door?" "Yes we can!" students reply. We ask a student to try this and they demonstrate that, in fact, they are able to walk out the door. But students become less certain when we think of the question this way: in order to get to the door it is also true that we first have to travel half the distance to the door; then half the remaining distance; then half the remaining distance; and so on and on. When I taught high school calculus, I would pose this problem to my class before we started working with limits. It is interesting that students found this an engaging problem and also one where they could not reach consensus. It seems that if you do not think about it, you can easily walk out the door. But when you do think of the infinite fractional distances to be covered, the task becomes less certain. This should not be surprising as the "walk out the door" problem is a variation of Zeno's paradoxes, which have puzzled mathematicians and philosophers for over two millennia. As Tall and Tirosh (2001) have noted, "infinity has fascinated mankind since time immemorial" (p. 129).

In the Grade 3 classroom, we also read and discuss a retelling of the story of Rapunzel, entitled *To Infinity and Beyond* (Gadanidis & Gadanidis, 2011). In this version of the classic fairytale, when the prince climbs through the window he notices that the cell door is open. "Rapunzel, why didn't you just walk out the door?" he asks, and Rapunzel tells him about her mathematical dilemma of having to pass through an infinite number of fractional distances. Mathematical turning points in stories (such as the Prince noticing the cell door being open and Rapunzel relating her mathematical dilemma) are acted out in the classroom and other parts of the story are performed as choral readings, thus actively engaging students in the storied mathematical experience. The story is also paused at such turning points and students are encouraged to suggest what might come next or how the dilemma might be resolved.

In our class discussions, students identify infinity as the stumbling block to "walking out the door." If they had to cross one hundred or one thousand or even a million fractional distances, it would seem possible. But with an infinite number of fractional distances it becomes a puzzle. So we ask, "How big is infinity? Would it fit in the gym? Would it fit in our classroom? Can you hold it in your hand?" The typical answers are "No" or "Yes, but it would eventually spill over." Some students suggest writing the word infinity or the symbol for infinity (which they encounter in the story we read) on a piece of paper, and holding it in your hand.

Students, working in pairs, are given a set of five square pieces of paper, all the same size, and all having a 16×16 square grid on them. On the first square, they shade in a representation of the fraction one half. On the second square they shade in one quarter. Then one eighth, one sixteenth and one thirty-second. We ask them to imagine doing this with more and more squares, on and on, forever. Then students discuss and decide in their pairs how big an area would be covered if we combined all the shaded parts. "Would the shaded area fit on the floor of our gym, for example?" Often students initially suggest that the shaded area would have to be quite large, maybe even infinite in size, since we keep adding to it without stop. Usually at least one student notices that if we use scissors to cut out the shaded fractions, they would all fit inside one of the squares. Taking up this idea, students start with a new square and shade in one half, then one half of the un-shaded part, and one half of the un-shaded part, and so on and on (see Figure 2). We explore different ways this might be done, leading to a variety of colourful representations, as shown in Figure 3. It is interesting that when the teacher then writes on the board "1/2 + 1/4 + 1/8 + 1/16 + 1/32 + ... = ?" and suggests that the sum must be infinitely large because we never stop adding numbers, Grade 3 students do not agree. Although there is some uncertainty at first and some discussion, students quite quickly convince one another and come to the consensus that the answer is either 1 or something very close to 1, since the

Figure 1. *Artistic rendering of classroom experience of "Infinity in my hand" by Ann Langeman.*

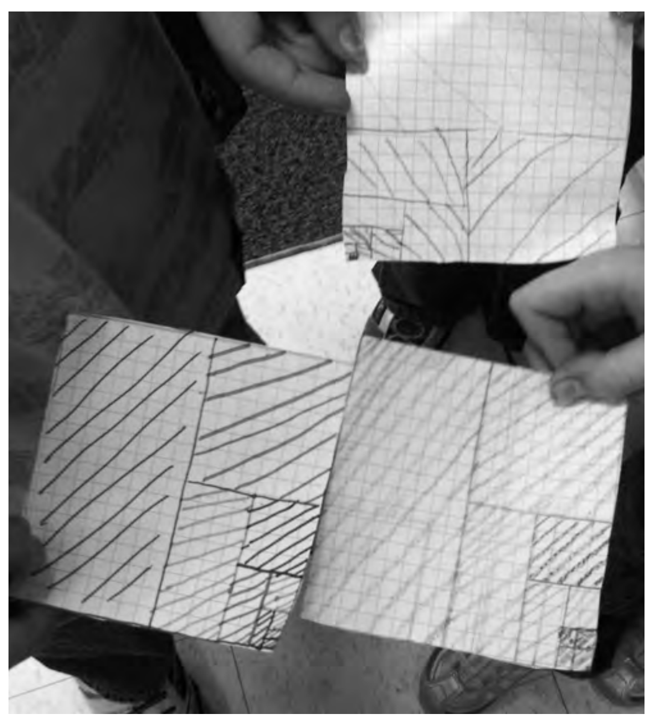

Figure 2. *Infinity in a square.*

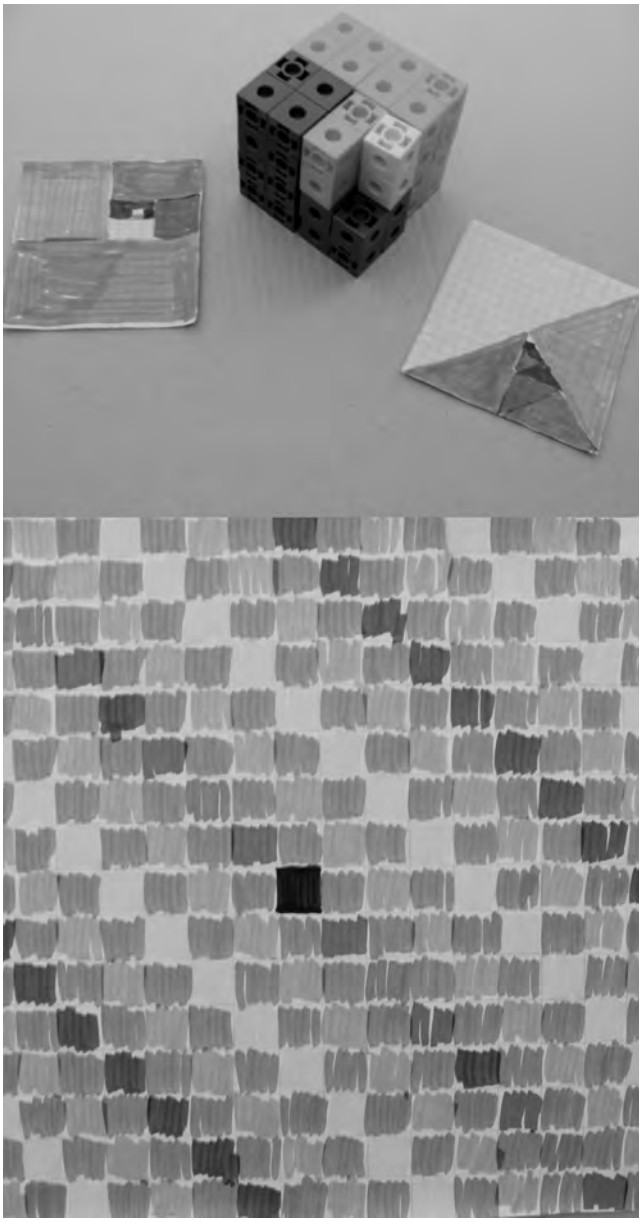

Figure 3. Various representations of infinity in a square and a cube.

I Like Infinity
I like infinity, I can hold it in my hands, it goes on forever
I like infinity, I can hold it in my hands, I can fold it up small
1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128
We love infinity!

Figure 4. "I like infinity" song by Grade 3 students.

shaded representations of these fractions fit in "one whole" (as shown in Figures 2 and 3). Since, the fractions never escape the square, the sum cannot be more than 1.

This activity is made accessible to Grade 3 students by designing representations with a low mathematical floor—shading fractions in a 16 ×16 square grid and depicting them as physical distances to the classroom door—while offering a high mathematical ceiling through attention to the "senior secondary school" concepts of infinity and limit. The mathematical twist in the story setting, and the acting out of the scene where Rapunzel explains why she cannot walk out the open door, draws students' thinking to the mathematics puzzle to be solved in a way that is both familiar and surprising. While making sense of the puzzles of "walking out the door" or "holding infinity in your hand" requires an early understanding of infinity and limit (high ceiling), the visual reasoning explored is comprehensible with little prerequisite mathematical knowledge (low floor). In the sequence of shading tasks, students learn how to represent the fractions 1/2, 1/4, 1/8 and so on as area diagrams. They notice relationships: there are two quarters in one half, two eighths in one quarter, four eighths in one half, and so on. However, once set in the context of infinity and limit, students also learn to share their Grade 3 *content* in the *context* of a deeply mathematical puzzle, such as "Mom, do you think I could hold infinity in my hand?" or "Dad, do you think I can walk out the living room door?" Students learn to represent fractions in different forms—but they also learn that mathematics is a subject of surprise and beauty that can be shared with family and friends.

Zwicky (2003, p. 38) states that "geometrical representations" such as those shown in Figures 2 and 3 attract our attention, and say: "look at things like this". They also lead to "understanding" through "seeing as" (p. 3). There is an aesthetic sense of pattern, of fit, of beauty when the infinite set of fractions are "seen as" fitting within a square. Taylor (2009) notes "the more beautiful something is, the more true it's likely to be."

At this stage, students work in small groups to record on chart paper what they have learned, using words, symbols, diagrams and pictures, and then they share their ideas with the rest of the class. As a culminating activity, in some cases, students work in small groups to author poems that summarize their learning, as shown in Figure 4, and they perform them as songs. In other cases, students script share-at-home mathematical dialogues that capture and share mathematical ideas and experiences. The song "Infinity in my hand" shown in Figure 5 (overleaf) [5] uses excerpts from dialogues scripted by Grade 3 students to share about infinity with their family and friends. Notice that students have created scenarios where mathematics is used to disrupt everyday events, such as taking out the trash or going to a movie, and offer the potential for mathematical surprise and insight. The song writing process typically involves the teacher creating an electronic document of all student writing, organizing ideas into themes or stanzas, and deleting repetitive statements and excess language while keeping statements made by as many different students as possible, so students see themselves in the song.

We have also used the infinity activity as a context for

Infinity in my hand

Hey Serena, do your chore
Take the garbage out the door
I can't Daddy, I learned in math
That I can't do it any more

It can't be true Serena
Your teacher is crazy
You can walk out the door
You've done it many times before

Halfway there Dad
and half of what's left
And half of what's left
it never ends
Infinity gets in the way
I heard my teacher say

Infinity gets in the way
I heard my teacher say
Infinity is so big
Infinity takes forever
But is there a way around it?
Can we think of something clever?

Let's go to a movie instead
Let's go Daddy, let's go ahead
I'd like to Serena, but too bad
We can't do it your teacher said

Halfway there
and half of what's left
And half of what's left
it never ends
Infinity gets in the way
I heard you say

I have an idea, a plan
Maybe Daddy we can
Infinity may be really big
But I can hold it in my hand

Infinity gets in the way
I heard my teacher say
Infinity is so big
Infinity takes forever
But there is a way around it
If we think of something clever

Take a square and shade a half
Then shade half of a half
On and on and on and on
Always shading half

Infinity in a square
Infinity hiding in there
It's a math surprise Serena
That is so much fun to share

Infinity gets in the way
I heard my teacher say
Infinity is so big
Infinity takes forever
But there is a way around it
If we think of something
clever

Half of a square
And a quarter and an eighth
On and on and on and on
The fractions make a whole

So we can do it Daddy
We can go to the movie
But first Serena do your chore
Take the garbage out the door

Infinity is not in the way
This is what I say
Infinity is so big
Infinity takes forever
But there is a way around it
If we think of something
clever

Figure 5. "Infinity in my hand" song.

Grade 4 students studying linear measurement. Working in small groups, students place a 200 centimetre long strip of masking tape vertically on a classroom or hallway wall, so that the masking tape reaches all the way to the floor. They drop a ball from the 200 centimetre height and mark on the masking tape the height the ball bounces. Then they drop the ball from the height it bounced and mark the new height bounced. They repeat this process one more time and calculate the total distance traveled by the ball after three bounces. They then study and extrapolate the pattern to estimate the total distance the ball would travel by the time it stops bouncing. Share-at-home dialogues students scripted were used to make the song "To infinity and beyond", an excerpt of which is shown in Figure 6. Notice that the lyrics contain the question students investigated, their initial estimates (100 m or even 1 km), and the surprising finding that the distance travelled is closer to 10 m (with the estimate varying depending on the type of ball used). [6]

Taking the topics of infinity and limit, which are typically studied in the senior years of secondary school, and making them accessible to students in younger grades, enriches the repertoire for understanding and representing underlying relationships, for both students and for teachers. The visual and concrete representations of infinity and limit would also benefit secondary school students, where in my experience they are currently rarely used. The surprises that an infinite set of numbers can have a finite sum and that infinity can be held in our hands draw our attention and they give us pleasure. Like watching a movie and predicting what might happen next, we get more pleasure when our guesses are wrong (for the right reasons) than when our guesses are correct (and the movie is predictable and boring) (Boorstin, 1990). Watson and Mason (2007) see mathematics as "an endless source of surprise, which excites us and motivates us. [...] The challenge is to create conditions for learners so that they too will experience a surprise" (p. 4). Mathematical

Hey Mom, look at this ball bounce
I'll drop it from a height of 2 metres
How far will it travel
Before it stops bouncing?
Going up and going down
Will it be 100 metres
or even a kilometre?
It keeps bouncing and bouncing
It must travel far
It's closer to ……. 10 metres Mom.

Figure 6. Excerpt from "To infinity and beyond" song.

surprises (at least surprises that are deeply and not superficially mathematical) require complex mathematical ideas and deep conceptual relationships, which is what the infinity and limit activity provides. In contrast, a typical worksheet that asks Grade 3 students to visually represent a list of unrelated fractions by shading in parts of diagrams offers no mathematical surprise and little mathematical pleasure.

Disrupting "mathematics" and "mathematician"

School mathematics needs to change. It must become more interesting, more challenging, more worthy of children's mathematical potential. It must also be an experience where students are seen as young mathematicians. As Ginsburg (2002) suggests, "children possess greater competence and interest in mathematics than we ordinarily recognize" and we should aim to develop a curriculum for them in which they are challenged to understand big mathematical ideas and have opportunities to "achieve the fulfillment and enjoyment of their intellectual interest" (p. 7). One way to do what Ginsburg suggests is to create a wider audience for mathematics and for the experiences and thinking of young mathematicians. The activities described above expand the sense of audience in four important ways.

First, grade-specific boundaries are disrupted, with ideas of limit and infinity stretched to be accessible across a broad range of grades. I believe that the ideas shared in this article only scratch the surface of the mathematics topics that may be addressed in a similar fashion. As a minimum, this low floor, high ceiling stretching helps engage young students with interesting and challenging mathematical ideas that may serve as *context* for learning existing *content* that may be less interesting and challenging mathematically.

Second, classroom boundaries are disrupted, with young students sharing their learning in ways that offer mathematical surprise and insight to a broader audience, including students in other classrooms, friends and family, and the wider world (through new media communication tools, such as the Math Performance Festival [2]). Opening such windows and becoming visible to the world beyond classroom walls adds purpose to learning and increases student motivation. It may also add a sense of pedagogical ownership and accountability that potentially increases teacher reflection.

Third, the notion of who is a mathematician and how a mathematician communicates is disrupted, with young students sharing stories of their personal mathematics experiences as a form of community service, offering mathematical surprise and insight about "secondary school" mathematical ideas to adults who possibly did not fully understand these ideas in their schooling. This is an important role-reversal from the more typical situation where students are consumers rather than producers or disseminators of mathematical knowledge.

Fourth, it disrupts existing notions of mathematics communication, with students being expected to share *good* stories about the mathematics they are learning, rather than simply describe the thinking process they follow to solve problems. The use of story as mathematical communication helps humanize mathematics and mathematicians. Writing about statistics, Paulos (2005) suggests that our "denial of the mutual dependence of stories and statistics, and the pedagogy that is a consequence of this denial, is one reason for the disesteem in which statistics, and mathematics and science generally, are widely held" (p. 4).

The four disruptive elements discussed above are intertwined. Good stories thrive on a sense of audience. Audiences demand stories that offer new, wonderful and surprising ways of looking at the world (of which mathematics is an integral part). The mathematically new, wonderful and surprising is made possible by taking high school ideas into the unfamiliar world of the elementary school student. Audiences also demand the pleasure of vicarious emotional experience, which students as young mathematicians provide as the lead agents or actors in the classroom mathematical dramas and songs they share as community service.

The approach shared in this article has been tested in several classrooms, in five project schools, over a period of five years. In one of the schools, the principal initially asked us to avoid grades that participate in mandatory provincial testing, fearing that a research project might negatively affect test results. However, in subsequent years, after witnessing the classroom activities, the principal asked us to include grades that were provincially tested. Although teachers in the project are committed to meeting mandated curriculum expectations and do not feel they have the time to engage students in activities that do not help them meet these expectations, year after year we are invited back to their classrooms. They appreciate that the low floor, high ceiling activities we collaboratively plan provide a rich mathematics context for the mathematics content they need to teach. In addition, the focus on arts-based communication helps teachers integrate mathematics with language arts, music, art, drama and media literacy, and meet mandated expectations in these areas in a mathematics context.

Artistic puzzles in mathematical storytelling

One way to frame the approach discussed in this article is as a focus on making mathematical connections (across grades), using multiple mathematical representations (as we transform and represent ideas to be easily accessible yet potentially complex), and making cross-curricular connections (through arts-based communication), all of which are pedagogical directions already sanctioned by recent curriculum documents and mathematics education reform initiatives. Seen in this light, why do we not see more classroom activities like the ones described in this article? One reason is that our curriculum focus on grade-specific content and our reliance on theories of what children cannot do at certain stages of development take attention away from the potential of children's mathematical minds. Another reason is that it is not a simple task to organize mathematical ideas based on principles of storytelling, so as to offer experiences of new and wonderful perspectives that help flex our imaginations, surprise, offer emotional moments, and visceral sensations of mathematical pattern and beauty.

Story is not a frill that we can set aside just because we have developed a cultural pattern of ignoring it in mathematics. Story is a biological necessity, an evolutionary adaptation that "train(s) us to explore possibility as well as actuality, effortlessly and even playfully, and that capacity makes all the difference" (Boyd, 2009, p. 188). Story makes us human and adds humanity to mathematics. Boyd (2001) notes that good storytelling involves solving *artistic* puzzles of how to create situations where the audience experiences the pleasure of surprise and insight. Solving such artistic puzzles may not be commonplace in mathematics pedagogy and it may not always be easy, but the mathematical beauty that results gives so much pleasure.

Notes

[1] The symposium was sponsored by the Fields Institute, Toronto, and was on *Online Mathematical Investigation as a Narrative Experience* (Gadanidis, Higginson & Sedig, 2005).

[2] The research and dissemination of the work reported in this article were funded by the Social Sciences and Humanities Research Council, the Knowledge Network for Applied Educational Research, the Fields Institute and the Imperial Oil Foundation.

[3] The Math Performance Festival offers students opportunities to share their mathematical performances with the public (see www.mathfest.ca) with funding from the Fields Institute, the Canadian Mathematical Society, the Imperial Oil Foundation, and Western University.

[4] With funding from the Fields Institute, Toronto–see www.JoyofX.ca

[5] The song is available at www.researchideas.ca

[6] The student performance of the song is available at www.edu.uwo.ca/mpc/mpf2010/mpf2010-131.html.

References

Boero, P., Douek, N. & Garuti, R. (2003) Children's conceptions of infinity of numbers in a fifth grade classroom discussion context. In Pateman, N.A., Dougherty, B.J. & Zilliox, J.T. (Eds.) *Proceedings of the 27th International Group for the Psychology of Mathematics Education Conference,* vol. 2, pp. 121-128. University of Honolulu, HI: PME.

Boorstin, J. (1990) *The Hollywood Eye: What Makes Movies Work.* New York, NY: Harper Collins Publishers.

Boyd, B. (2001) The origin of stories: Horton hears a Who. *Philosophy and Literature* **25**(2), 197-214.

Boyd, B. (2009) *On the Origin of Stories: Evolution, Cognition, and Fiction.* Cambridge, MA: Belknap Press of Harvard University Press.

Burton, L. (1999) The implications of a narrative approach to the learning of mathematics. In Burton, L. (Ed.) *Learning Mathematics: From Hierarchies to Networks,* pp. 21-35. New York, NY: Falmer Press.

Cummins, J., Brown, K. & Sayers, D. (2007) *Literacy, Technology, and Diversity: Teaching for Success in Changing Times*. Boston, MA: Allyn & Bacon.

Dissanayake, E. (1992) *Homo Aestheticus: Where Art Comes From and Why.* New York, NY: Free Press.

Egan, K. (2002) *Getting it Wrong from the Beginning: Our Progressive Inheritance from Herbert Spencer, John Dewey, and Jean Piaget.* New Haven, CT: Yale University Press.

Ernest, P. (1996) Popularization: myths, mass media and modernism. In Bishop, A.J., Clements, K., Keitel, C., Kilpatrick, J. & Laborde, C. (Eds.) *International Handbook of Mathematics Education,* pp.785-817. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Fernández-Armesto, F. (1997) *Truth: A History and a Guide for the Perplexed.* London, UK: Bantam Press.

Gadanidis, G. & Gadanidis, M. (2011) *To Infinity and Beyond: the Real story of Rapunzel.* Whitby, ON: Brainy Day Publications.

Gadanidis, G. & Hughes, J.M. (2011) Performing big math ideas across the grades. *Teaching Children Mathematics* **17**(8), 486-496.

Gadanidis, G., Higginson, W. & Sedig, K. (Eds.) (2005) *Online mathematical investigation as a narrative experience: Proceedings of a Fields Institute Symposium on Mathematics Through the Lenses of Art and Technology.* Faculty of Education, University of Western Ontario.

Ginsburg, H. P. (2002) Little children, big mathematics: learning and teaching in the pre-school. In Cockburn, A.D. & Nardi, E. (Eds.) *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education,* vol. 1, pp. 3-14. Norwich, UK: PME.

Henderson, D. (1981) Three papers: mathematics and liberation, Sue is a mathematician, mathematics as imagination. *For the Learning of Mathematics* **1**(3), 12-15.

Higginson, W. (2008) *Interview*. Available at www.edu.uwo.ca/dmp/Higginson.

Hughes, J. & Gadanidis, G. (2010) Learning as community service: thinking with new media. *Journal of Educational Multimedia and Hypermedia* **19**(3), 287-306.

Hull, G.A. & Katz, M-L. (2006) Crafting an agentive self: case studies of digital storytelling. *Research in the Teaching of English* **41**(1), 43-81.

Kamii, C. & Ewing, J.K. (1996) Basing teaching on Piaget's constructivism. *Childhood Education* **72**(5), 260-264.

Kleiner, I. (2001) History of the infinitely small and the infinitely large in calculus. *Educational Studies in Mathematics* **48**(2), 137-174.

Kline, M. (1980). *Mathematics: The Loss of Certainty.* New York, NY: Oxford University Press.

Miller, P. (1994) Narrative practices: their role in socialization and self-construction. In Neisser, U. & Fivush, R. (Eds.) *The Remembering Self: Construction and Agency in Self Narrative,* pp. 158-179. Cambridge, UK: Cambridge University Press.

Moreau, M-P., Mendick, H. & Epstein, D. (2010) Constructions of mathematicians in popular culture and learners' narratives: a study of mathematical and non-mathematical subjectivities. *Cambridge Journal of Education* **40**(1), 25-38.

Papert, S. (1980) *Mindstorms: Children, Computers, and Powerful Ideas.* New York, NY: Basic Books.

Piaget, J. (2008). Intellectual evolution from adolescence to adulthood. *Human Development* **51**(1), 40-47. (Original work published 1972).

Paulos, J.A. (2005) A mathematician explores the gap between stories and statistics, logic and language. Paper presented at the Mathematics and Narrative conference, Mykonos, Greece. July. Retrieved 7/9/2011 from thalesandfriends.org.

Piatek-Jimenez, K. (2008) Images of mathematicians: a new perspective on the shortage of women in mathematical careers. *ZDM* **40**(4), 633-646.

Picker, S.H. & Berry, J.S. (2000) Investigating pupils' images of mathematicians. *Educational Studies in Mathematics* **43**(1), 65-94.

Tall, D. & Tirosh, D. (2001) Infinity — the never-ending struggle. *Educational Studies in Mathematics* **48**(2-3), 129-136.

Taylor, P. (2009) Telescopes and paraboloids. Toronto, ON: The Fields Institute. Available at www.fields.utoronto.ca/mathwindows.

Walkerdine, V. (1984) Development psychology and the child-centred pedagogy: the insertion of Piaget into early education. In Henriques, J., Hollway, W., Urwin, C., Venn, C. & Walkerdine, V. (Eds.) *Changing the Subject: Psychology, Social Regulation and Subjectivity,* pp. 153-202. London, UK: Methuen & Co. Ltd.

Watson, A. & Mason, J. (2007) Surprise and inspiration. *Mathematics Teaching* **200**, 4-7.

Whitin, D.J. & Gary, C.C. (1994) Promoting mathematical explorations through children's literature. *The Arithmetic Teacher* **41**(7), 394-399.

Whitin, D.J. & Wilde, S. (1995) *It's the Story that Counts: More Children's Books for Mathematical Learning, K-6.* Portsmouth, NH: Heinemann Educational Books.

Zwicky, J. (2003) *Wisdom & Metaphor.* Kentville, NS: Gaspereau Press.