

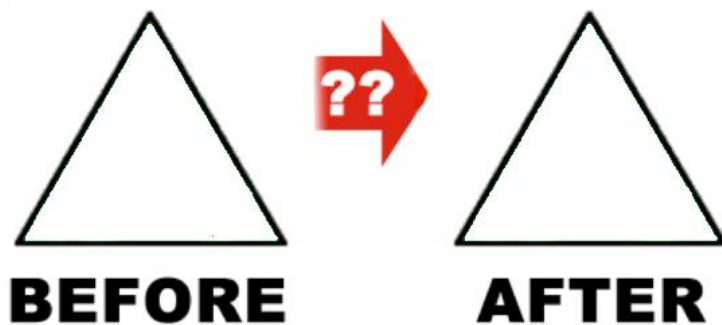
SYMMETRY v.5 (triangle)



What is “symmetry”? I can’t remember. Help!

TRIANGLE PUZZLE

My teacher loves puzzles—I do too!
Here is the latest one: “How did the triangle change?”

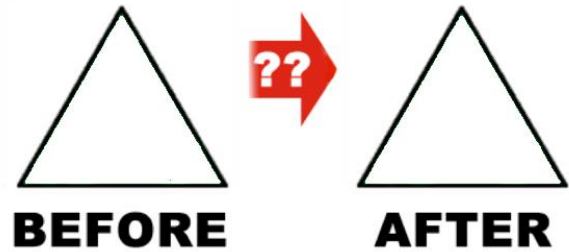


I don’t see a change. Do you have any ideas?

SYMMETRY DEFINITION

Did you know mathematicians define symmetry as a transformation?

A symmetry is a transformation that leaves an object looking unchanged.

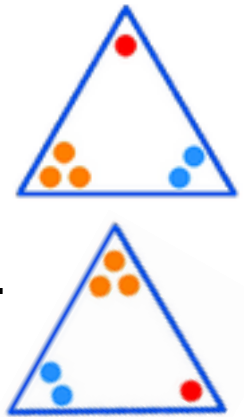


Now I get the puzzle!

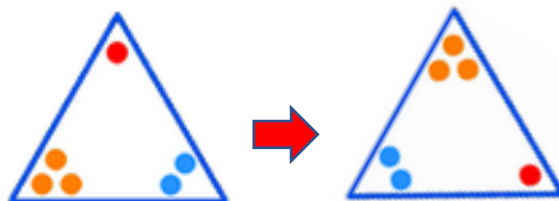
TRIANGLE ROTATION SYMMETRIES

I have an idea! Let's label the vertices of the triangle with 1, 2 and 3 dots, to better see the transformations.

Rotate the triangle $1/3$ turn (or 120 degrees) clockwise. The order of the dots will change: **123** to **312**.



I wonder how many **rotation symmetries** a triangle has. My guess is 3, since it has 3 sides. Am I right? Here's one of them. Can you find the rest?



Rotate $1/3$ or 120 degrees



Rotate _____ or _____ degrees

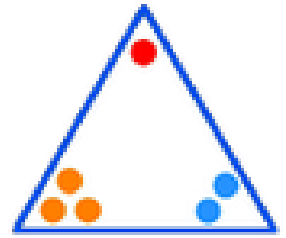


Rotate _____ or _____ degrees

PERMUTATIONS

I learned a new math word: **permutation!**

A permutation is an arrangement.



Rotating the triangle, we discovered 3 permutations of the numbers 1, 2 & 3: **123**, **312** & **231**.

But there are more ways to arrange these numbers—right?

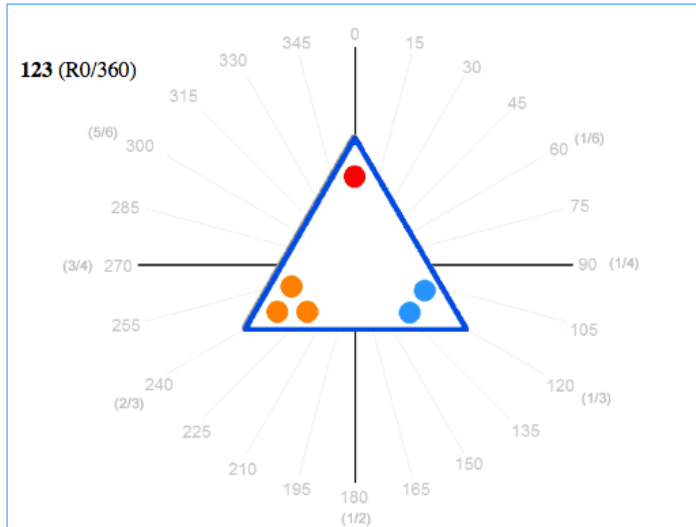
Help me find them—it will be fun!

Use an organized list, so you don't miss any—do you agree?


Then, circle the 3 rotation permutations for the triangle.

ROTATE TRIANGLE WITH CODE

Wow! We can use coding for transformations! Want to try it?
Go to researchideas.ca/sym




TIP: If this view does not fit on your screen, press Control (or Command) and minus (-) to reduce screen resolution.

Select the triangle.
Enter this code. 
Then click Run Code.

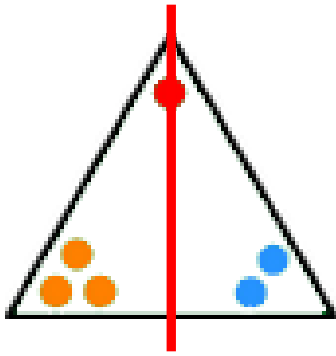
Notice the rotation permutations listed on the left of the screen.

Is it possible to get permutations other than **123**, **312** & **231** by rotating the triangle? What do you think?

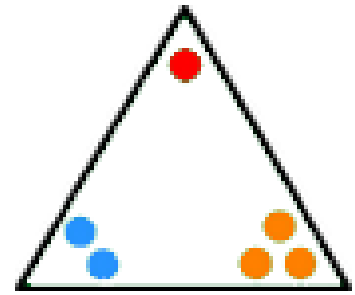
TRIANGLE REFLECTION SYMMETRIES

Did you know that “flip” and “reflect” is the same transformation?

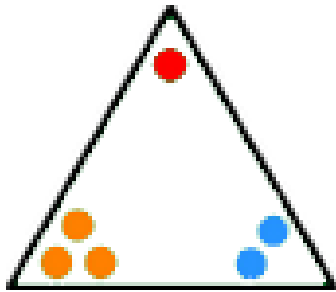
For example, we can flip, or reflect, the triangle across its vertical line of symmetry. Do you see how the permutation changes?



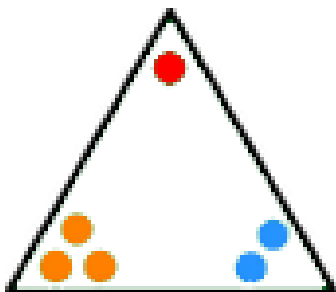
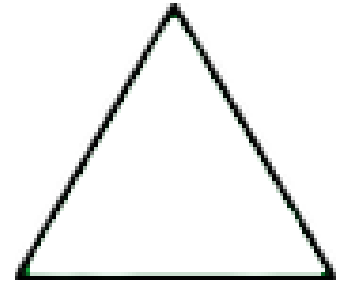
123 ⇒ _____



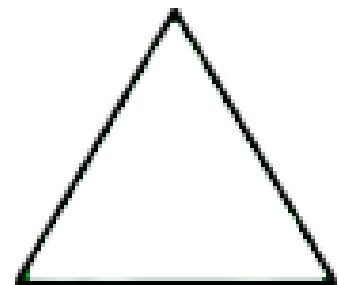
I think there are 2 more ways. Help me draw the remaining lines of symmetry and record the permutations.



123 ⇒ _____

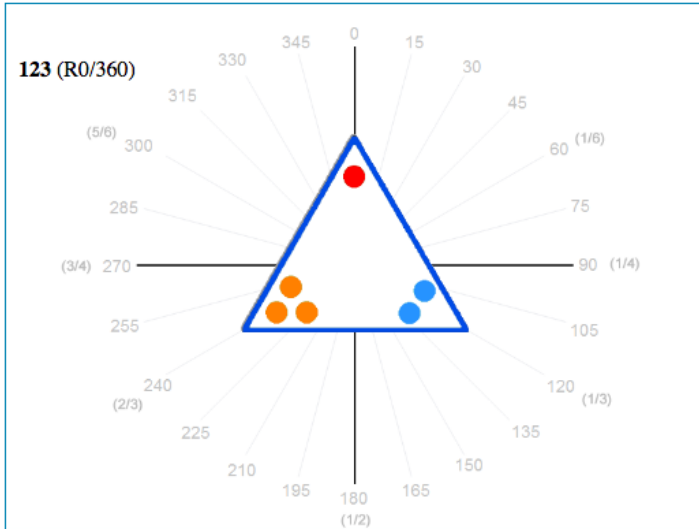


123 ⇒ _____



REFLECT TRIANGLE WITH CODE

Hey! We can use code for reflections too! Let's try it!
Go to researchideas.ca/sym




Select the triangle. Enter this code.
Then click Run Code.

The **0 degree line** is the vertical line of symmetry. What are the degrees for the other lines of symmetry?

For each line of symmetry, set the degrees in the code, and then Run Code. Record the permutation for each reflection.
Do they match the permutations you found on the previous page?

On page 3, underline permutations that match the reflection symmetries of the triangle.

COMBINE TRIANGLE REFLECTIONS WITH CODE

I made a great discovery!
You'll be amazed!

Go to researchideas.ca/sym
Enter and run this code.

Do you see what I discovered?
We flip the triangle across
the 0 degree line.
Then we flip it across
the 60 degree line.
And the permutation is **312**!

```

reset shape
flip across 0 degree line
wait 1 seconds
flip across 60 degree line
  
```

Do you see why this is amazing? Need a tip?
Go to page 3 and see if **312** is circled or underlined.

Let's try more combinations.
Change **0** and **60** degrees to any pair of the following:

0, 60, 120, 180, 240,
300, 360, -60, -120,
-180, -240, 300, 360

Repeat with different pairs.

Also, try 3 flips in a row!

What do you notice?

Any surprises?

```

reset shape
flip across 0 degree line
wait 1 seconds
flip across 60 degree line
wait 1 seconds
flip across 120 degree line
  
```

ROTATE SQUARE WITH CODE

Let's rotate the square too!

We discovered that the triangle has 3 rotation symmetries, with permutations 123, 312 & 231.

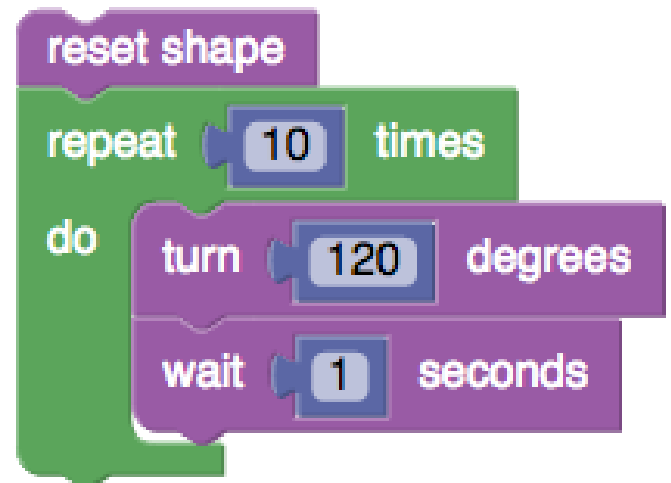


What about the square? Help me predict:

- How many rotation symmetries does the square have? _____
- What are the permutations? 1234, _____, _____, _____

O.K. Let's code. Select the square in the coding environment.

This is the code we used to rotate the triangle. Hmm. How many degrees should we rotate the square, so that it looks unchanged?



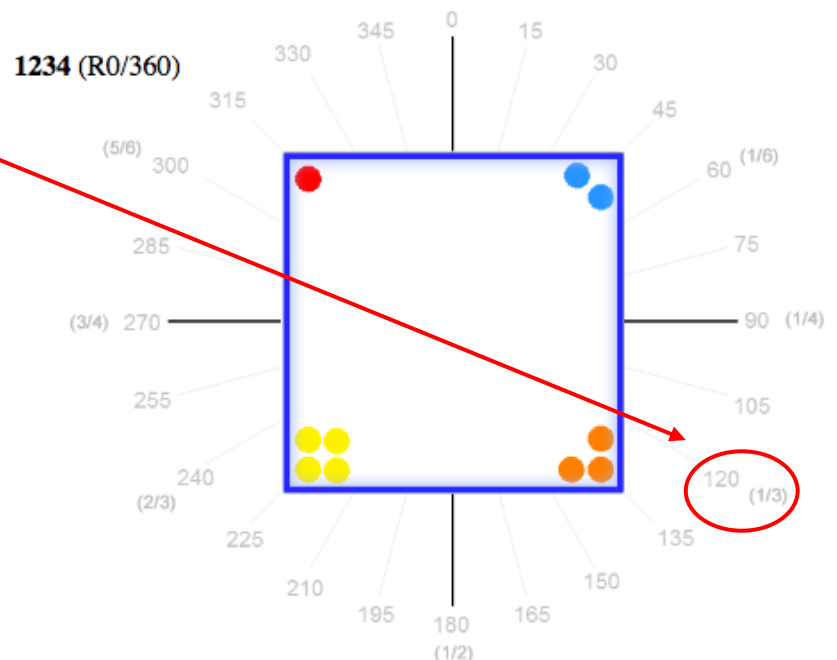
Stuck?

Look at the fraction turns.

For the triangle, it was $1/3$ turn, or 120 degrees.

Should the square rotate $1/6$, $1/4$ or $1/3$ of a turn?


What's the matching degree measurement?



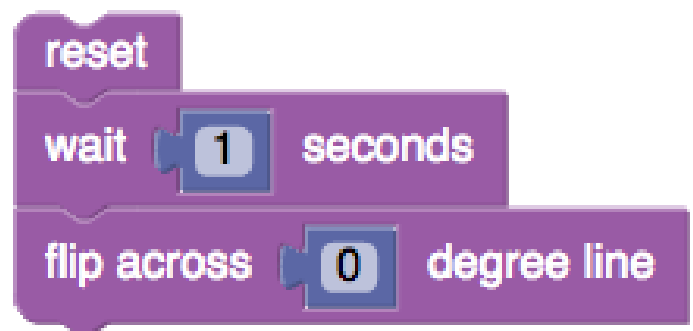
PERMUTATIONS

I wonder: there were 6 permutations with the numbers 1, 2 & 3. How many permutations are there with the numbers 1, 2, 3 & 4? Can you help me find them? Of course you can!

REFLECT SQUARE WITH CODE

Let's try this with code! 
Go to researchideas.ca/sym
Select the square.

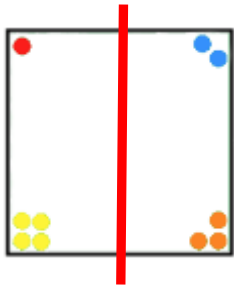
The **0 degree line** is the vertical line of symmetry.



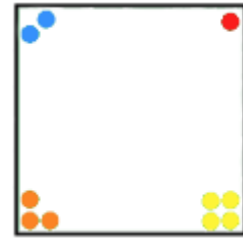
What are the degrees for the other lines of symmetry?

SQUARE REFLECTION SYMMETRIES

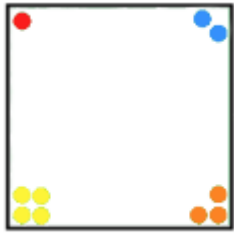
What are the reflection symmetries of the square?
Here's one. Do you see how the permutation changes?



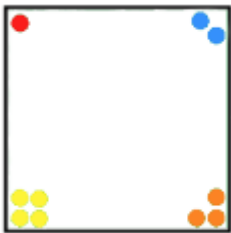
1234 ⇒ _____



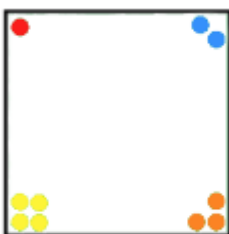
Three more, I think. Draw lines of symmetry & record permutations.



1234 ⇒ _____



1234 ⇒ _____



1234 ⇒ _____



MATH DISCOVERY

We made an important math discovery! Don't you agree?

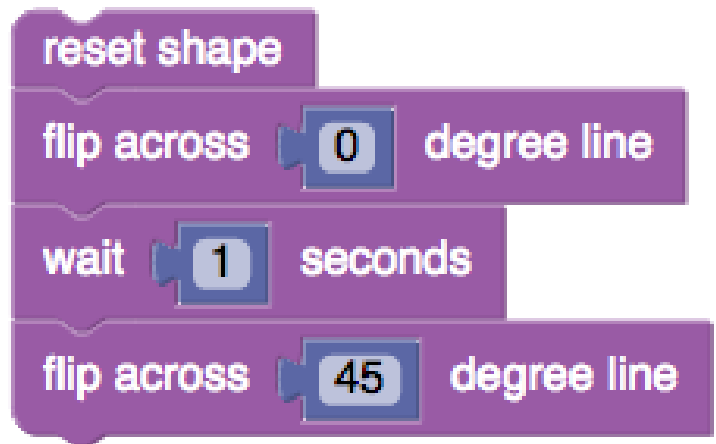
For triangles, rotations are a different *species* than the reflections:

- When we combine rotations, we always get another rotation.
- But, when we combine reflections, sometimes we get a reflection and sometimes a rotation.

COMBINE SQUARE REFLECTIONS WITH CODE

I bet you're wondering, "What about the square?"
Me too!

Let's combine square reflections.



MORE SHAPES

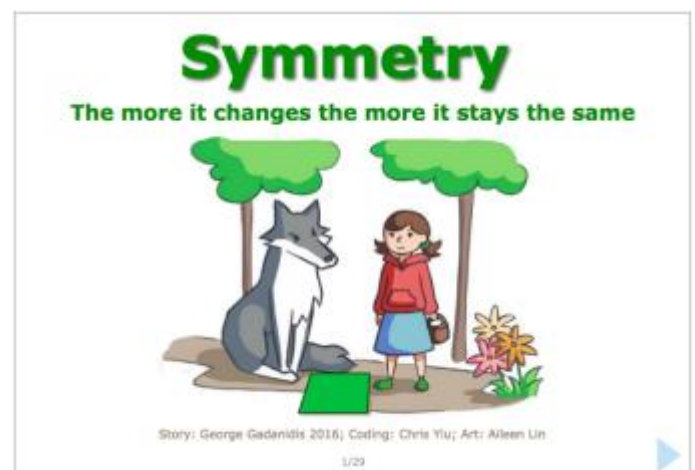
Let's not forget the hexagon!
And the other shapes, too!
What are their symmetries?
What happens when we combine them?



MATH STORY

Read a symmetry story at:
researchideas.ca/sym/story.html

*Symmetry: the more it changes
the more it stays the same*



REFLECT

I learned so much! How about you?

What did you learn?

What else do you want to know or explore?